JHEP HYPER VERSION

Received: *September 5, 2001*, Accepted: *October 29, 2001*
Revised: *October 29, 2001*

Effects of quenching and partial quenching on penguin matrix elements

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JHEP10(2001)037

Abstract: In the calculation of non-leptonic weak decay rates, a "mismatch" arises when the QCD evolution of the relevant weak hamiltonian down to hadronic scales is performed in unquenched QCD, but the hadronic matrix elements are then computed in (partially) quenched lattice QCD. This mismatch arises because the transformation properties of penguin operators under chiral symmetry change in the transition from unquenched to (partially) quenched QCD. Here we discuss QCD-penguin contributions to $\Delta S = 1$ matrix elements, and show that new low-energy constants contribute at leading order in chiral perturbation theory in this case. In the partially quenched case (in which sea quarks are present), these low-energy constants are related to electro-magnetic penguins, while in the quenched case (with no sea quarks) no such relation exists. As a simple example, we give explicit results for $K^+ \to \pi^+$ and $K^0 \to$ *vacuum* matrix elements, and discuss the implications for lattice determinations of $K \to \pi\pi$ amplitudes from these matrix elements.

Keywords: Kaon Physics, Lattice QCD, Chiral Lagrangians.

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1. Introduction

Strong and electro-magnetic penguin operators are an important part of the $\Delta S = 1$ weak hamiltonian at hadronic scales, in particular with respect to CP-violating kaon-decay amplitudes. In this paper, we will consider LR operators of the form

$$Q_{\text{penguin}} = (\overline{s}d)_L(\overline{q}Xq)_R\,, \tag{1.1}$$

where $q = u, d, s$, $X = \mathbf{1}$ for QCD penguins and $X = Q = \text{diag}(1, -1/2, -1/2)$ for electro-magnetic penguins. In eq. (1.1)

$$(\overline{q}_1 q_2)_{L,R} = \overline{q}_1\gamma_\mu P_{L,R}q_2\,, \tag{1.2}$$

with $P_{L,R} = (1 \mp \gamma_5)/2$ left- and right-handed projectors. For each X the color indices can be contracted in two ways, corresponding to the operators $Q_{5,6}$ for $X = \mathbf{1}$ and $Q_{7,8}$ for $X = Q$ [1]. Of course, since the strong and EM interactions conserve parity, LL operators with $P_R \to P_L$ in the second factor also occur; they can be written as linear combinations of the operators $\mathcal{O}_{1-4}$ (see e.g. ref. [2] for a list of all those operators).

In order to calculate the penguin contribution to non-leptonic kaon decays, one may employ Lattice QCD in order to obtain the non-perturbative part, while the perturbative part is encapsulated in the Wilson coefficients, and can be calculated using perturbative QCD in the continuum [3]. The lattice part is typically done in the quenched approximation [4], in which the fermion determinant is replaced by a constant. This amounts to ignoring all sea-quark effects. When, in the future, sea quarks will be included, one still may wish to use valence- and sea-quark masses which are not equal to each other, a situation known as "partial quenching."

Partially quenched QCD (PQQCD) can be systematically understood in a lagrangian framework by coupling the gluons to three sets of quarks [5]: a set of K valence quarks q_{vi} with masses $m_{v1}, m_{v2}, \ldots, m_{vK}$, a set of N sea quarks q_{si} with masses $m_{s1}, m_{s2}, \ldots, m_{sN}$, and a set of K ghost quarks q_{gi} with masses $m_{v1}, m_{v2}, \ldots, m_{vK}$. The ghost quarks are identical to the valence quarks, except for their statistics, which is chosen to be bosonic [6]. Hence their determinant cancels that from the valence quarks (notice they have the same masses), thus justifying their name: all valence-quark loops coming from the fermion determinant are canceled by ghost-quark loops. Quenched QCD (QQCD) corresponds to the special case $N = 0$. Unquenched QCD below the charm-quark threshold corresponds to the choice $K = N = 3$ and $m_{si} = m_{vi}$, $i = 1, \ldots, 3$. In this case penguin operators are written as in eq. (1.1), with $m_{v1} = m_{\rm up}$, $m_{v2} = m_{\rm down}$, $m_{v3} = m_{\rm strange}$), but the analysis of this paper applies for arbitrary K and N.

The total number of quarks is thus $2K + N$, and correspondingly, the chiral symmetry group enlarges from $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ to the graded group $\mathrm{SU}(K{+}N|K)_L \times \mathrm{SU}(K{+}N|K)_R$, where the grading is a consequence of the fact that fermionic (valence or sea) quarks can be rotated into bosonic (ghost) quarks, and *vice versa* [7, 5]. A consequence of this is that the QCD penguins (for which $X = \mathbf{1}$ in eq. (1.1)) are no longer right-handed singlets after making the transition from unquenched to (partially) quenched QCD. We will show that this leads to important consequences for the interpretation of lattice results, with an argument based on chiral perturbation theory (ChPT). The point here is that the operators of eq. (1.1) are obtained by the unquenched QCD evolution of the weak operator from the weak scale $\sim M_W$ down to the hadronic scale $\sim m_c$, so that $(\overline{q}q) = (\overline{u}u) + (\overline{d}d) + (\overline{s}s)$ is only a singlet under flavor SU(3), but not under $\mathrm{SU}(K + N|K)$. In contrast, one could also imagine the situation in which strong interactions are quenched at all scales, in which case the QCD penguins would have taken the form

$$\begin{aligned} Q^{PQS}_{\rm penguin} &= (\overline{s}d)_L \left(\sum_{i \ \text{valence}} \overline{q}_{vi} q_{vi} + \sum_{i \ \text{sea}} \overline{q}_{si} q_{si} + \sum_{i \ \text{ghost}} \overline{q}_{gi} q_{gi} \right)_R \qquad (1.3) \\ &= \mathrm{str}\,(\Lambda \psi \overline{\psi} \gamma_\mu P_L)\ \mathrm{str}\,(\psi \overline{\psi} \gamma_\mu P_R)\,, \\ \Lambda_{ij} &= \delta_{i3} \delta_{j2}\,, \qquad (1.4) \end{aligned}$$

where $\psi = (q_v, q_s, q_g)$ and str is the supertrace, which arises because $q_{v,s}$ and $\overline{q}_{v,s}$ resp. q_g and $\overline{q}_g$ anti-commute resp. commute. These operators do transform as a singlet under the full PQ symmetry group $\mathrm{SU}(K{+}N|K)_R$. They have been discussed before [8, 9], but clearly, this analysis is not complete when one considers the weak hamiltonian for which the running from the weak scale to the hadronic scale has been calculated in unquenched QCD.

In this paper, we will consider the situation with operators of the form (1.1) instead of (1.3) at the hadronic scale. We postpone a more complete discussion, including also LL operators until later [10], because the LR case is somewhat simpler, and, more importantly, because the consequences for the interpretation of (partially) quenched lattice results are more dramatic in the LR case.

2. Penguins in (partially) quenched QCD

The QCD penguin operators, eq. (1.1) with $X = \mathbf{1}$, can be decomposed as

$$
\begin{aligned}
Q^{QCD}_{\rm penguin} &= \frac{K}{N}\,{\rm str}\,(\Lambda\psi\overline{\psi}\gamma_\mu P_L)\,{\rm str}\,(\psi\overline{\psi}\gamma_\mu P_R) + {\rm str}\,(\Lambda\psi\overline{\psi}\gamma_\mu P_L)\,{\rm str}\,(A\psi\overline{\psi}\gamma_\mu P_R)\,, \\
&\equiv \frac{K}{N}\,Q^{PQS}_{\rm penguin} + Q^{PQA}_{\rm penguin}\,, \qquad (2.1)\\
A &= {\rm diag}\left(1-\frac{K}{N},\dots,1-\frac{K}{N},-\frac{K}{N},\dots,-\frac{K}{N}\right), \qquad (2.2)
\end{aligned}
$$

where the first K (valence) entries of A are equal to $1 - K/N$, and the next $N + K$ (sea and ghost) entries are equal to $-K/N$. The superscripts PQS and PQA indicate that these operators transform in the singlet and adjoint representations of $\mathrm{SU}(K + N|K)_R$, respectively. It is clear that these operators cannot transform into each other from the fact that A is supertrace-less, while the unit matrix is not. It follows that in PQQCD the QCD penguin is a linear combination of two operators which transform in different irreducible representations (irreps) of the PQ symmetry group.

In fact, we may also embed the EM penguin $Q^{EM}_{\rm penguin}$ into PQQCD, by enlarging the charge matrix Q to $Q = {\rm diag}(1, -1/2, -1/2, 0, \dots, 0)$. Since Q is also supertrace-less, the EM penguin is also a component of the adjoint irrep, and $Q^{EM}_{\rm penguin}$ and $Q^{PQA}_{\rm penguin}$ are thus components of the same irrep.

In the quenched case, for which $N = 0$ (no sea quarks at all), the situation is special. The decomposition reads

$$
\begin{aligned}
Q^{QCD}_{\rm penguin} &= \frac{1}{2}\,{\rm str}\,(\Lambda\psi\overline{\psi}\gamma_\mu P_L)\,{\rm str}\,(\psi\overline{\psi}\gamma_\mu P_R) + {\rm str}\,(\Lambda\psi\overline{\psi}\gamma_\mu P_L)\,{\rm str}\,(\hat{N}\psi\overline{\psi}\gamma_\mu P_R)\,, \\
&\equiv \frac{1}{2}\,Q^{QS}_{\rm penguin} + Q^{QNS}_{\rm penguin}\,, \qquad (2.3)\\
\hat{N} &= \frac{1}{2}{\rm diag}(1,\dots,1,-1,\dots,-1)\,, \qquad (2.4)
\end{aligned}
$$

where the first K (valence) entries of $\hat{N}$ are equal to $\frac{1}{2}$, and the last K (ghost) entries are equal to $-\frac{1}{2}$. The first operator in the decomposition is a singlet, while the second is not, under $\mathrm{SU}(K|K)_R$ (NS for non-singlet). However, the unit matrix now has a vanishing supertrace, while $\hat{N}$ has not. It is easy to show that, while $Q^{QS}_{\rm penguin}$ obviously cannot transform into anything else, $Q^{QNS}_{\rm penguin}$ can transform into the singlet operator, so that the non-singlet operators do not form a representation by themselves. These group-theoretical facts correspond to the way these operators can mix under the strong interactions: $Q^{QS}_{\rm penguin}$ cannot mix into any other operator, but one can easily verify that $Q^{QNS}_{\rm penguin}$ can mix with $Q^{QS}_{\rm penguin}$ through penguin-like diagrams.

The situation is also different with respect to the EM penguins. Since the charge matrix Q is supertrace-less, it cannot rotate into $Q^{QNS}_{\rm penguin}$, and $Q^{EM}_{\rm penguin}$ and $Q^{QNS}_{\rm penguin}$ are not two components of the same irrep. Neither are $Q^{EM}_{\rm penguin}$ and $Q^{QS}_{\rm penguin}$, because $Q^{QS}_{\rm penguin}$ cannot be rotated into $Q^{EM}_{\rm penguin}$. We conclude that none of the three operators are related by being members of the same irrep in the quenched case. Note that this is unlike the PQ case, for which both non-singlet operators are members of the same irrep. The difference originates in the fact that the unit matrix is not supertrace-less for $N \neq 0$, but it is for $N = 0$.[1]

3. Representation of penguins in ChPT

It is well known (see e.g. ref. [2]) that the operators representing QCD penguins start at order p^2 in ChPT, while those representing EM penguins start at order p^0.[2] This follows from the fact that they transform differently under $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$: QCD penguins as $(8,1)$ and EM penguins as $(8,8)$. Denoting the adjoint representation of the PQ group by A, we found in the previous section that $Q^{PQS}_{\rm penguin}$ transform as $(A,1)$, while $Q^{PQA}_{\rm penguin}$ (and $Q^{EM}_{\rm penguin}$) transform as (A,A) under $\mathrm{SU}(K+N|K)_L \times \mathrm{SU}(K+N|K)_R$. It follows that, to lowest order in ChPT and in euclidean space, these operators are represented by[3]

$$Q^{PQS}_{\rm penguin} \to -\alpha_1^{(8,1)}\,\mathrm{str}\,(\Lambda L_\mu L_\mu) + \alpha_2^{(8,1)}\,\mathrm{str}\,(\Lambda X_+)\,, \tag{3.1}$$

$$Q^{PQA}_{\rm penguin} \to f^2\,\alpha^{(8,8)}\,\mathrm{str}\,(\Lambda \Sigma A \Sigma^\dagger)\,, \tag{3.2}$$

where

$$L_\mu = i\Sigma\partial_\mu\Sigma^\dagger\,, \qquad X_\pm = 2B_0(\Sigma M^\dagger \pm M\Sigma^\dagger)\,, \tag{3.3}$$

with M the quark-mass matrix, B_0 the parameter B_0 of ref. [12], $\Sigma = \exp(2i\Phi/f)$ the unitary field describing the partially-quenched Goldstone-meson multiplet, and f the bare pion-decay constant normalized such that $f_\pi = 132$ MeV. The low-energy constants (LECs) $\alpha^{(8,1)}_{1,2}$, introduced in ref. [11], would also appear in the unquenched theory, but the appearance of the LEC $\alpha^{(8,8)}$ is special to the PQ case. Nevertheless, it has a direct physical meaning, because the same LEC also appears in the bosonization of the EM penguin:

$$Q^{EM}_{\rm penguin} \to f^2\,\alpha^{(8,8)}\,\mathrm{str}\,(\Lambda\Sigma Q\Sigma^\dagger)\,, \tag{3.4}$$

[1] MG thanks Noam Shoresh for instructive discussions on this point.

[2] In standard ChPT power counting EM penguins are of order e^2p^0; here we are not concerned with the factor e^2.

[3] In ref. [9] we used the shorthand notation $\alpha^8_{1,2}$ for $\alpha^{(8,1)}_{1,2}$; here we follow the notation of ref. [11].

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because of the fact that the EM penguin $Q^{EM}_{\rm penguin}$ and the non-singlet PQ strong penguin $Q^{PQA}_{\rm penguin}$ are in the same irrep of the PQ symmetry group. In addition, taking the number of sea quarks in the PQ theory to be the same as in the real world, $N = 3$, the LECs are the same as those of the unquenched theory in the limit in which the η' decouples [13], thus justifying their names.[4]

As one would expect, the quenched case is different. First, all LECs are those of the $N = 0$ theory, and we do not know of any argument connecting them to those of the real world. Second, as pointed out in the previous section, $Q^{QNS}_{\rm penguin}$ and $Q^{EM}_{\rm penguin}$ do not belong to the same irrep, and their respective LECs are in principle different. We therefore get the following quenched bosonization rules, to leading order in ChPT:

$$\begin{aligned}
Q^{QS}_{\rm penguin} &\to -\alpha^{(8,1)}_{q1}\ {\rm str}\,(\Lambda L_\mu L_\mu) + \alpha^{(8,1)}_{q2}\ {\rm str}\,(\Lambda X_+)\,, \\
Q^{QNS}_{\rm penguin} &\to f^2\ \alpha^{NS}_q\ {\rm str}\,(\Lambda\Sigma\hat{N}\Sigma^\dagger)\,, \\
Q^{EM}_{\rm penguin} &\to f^2\ \alpha^{(8,8)}_q\ {\rm str}\,(\Lambda\Sigma Q\Sigma^\dagger)\,,
\end{aligned} \tag{3.5}$$

where the subscript q indicates that these are the LECs of the quenched theory. One concludes that in the quenched theory yet another LEC α^{NS}_q appears, with no counterpart in the PQ theory. Similar new LECs will also occur for LL operators, but, as we will see in the next section, they are particularly important in the LR case, because the non-singlet LR operators are order p^0 in ChPT, thus potentially leading to an enhancement relative to the unquenched case.

4. $K \to \pi$ and $K \to$ *vacuum* matrix elements to order p^2 in ChPT

We will now show how the new LECs come into play, by considering the simple example of $K \to \pi$ and $K \to$ *vacuum* ($K \to 0$) matrix elements, to leading order in ChPT. We should emphasize however, that the new contributions to QCD penguins found above are properties of these operators, not only of certain matrix elements. In particular, new contributions would also show up in direct $K \to \pi\pi$ matrix elements of $Q_{\rm penguin}$ with $X = \mathbf{1}$.

The new operators, $Q^{PQA}_{\rm penguin}$ and $Q^{QNS}_{\rm penguin}$, do not contribute to these matrix elements at lowest order in ChPT, i.e. at order p^0. However, they do in general contribute at order p^2. Since $Q^{(P)QS}_{\rm penguin}$ starts at order p^2, the new contributions from $Q^{PQA}_{\rm penguin}$ and $Q^{QNS}_{\rm penguin}$ compete at the *leading* order of the chiral expansion of these matrix elements, and will have to be taken into account even if one analyzes lattice results using only leading-order ChPT.

[4] This follows because the LECs of the PQ theory only depend on N, but not on quark masses.

These new contributions at order p^2 may originate from one-loop diagrams and from additional terms in the bosonization of $Q^{PQA}_{\rm penguin}$. Those relevant for the $K \to 0$ and $K \to \pi$ matrix elements are[5]

$$
\begin{aligned}
Q_1^{PQA} &= \frac{\beta_1^{(8,8)}}{(4\pi)^2}\,\mathrm{str}\left(\Lambda\{\Sigma A\Sigma^\dagger, L_\mu L_\mu\}\right) , \\
Q_2^{PQA} &= \frac{\beta_2^{(8,8)}}{(4\pi)^2}\,\mathrm{str}\left(\Lambda L_\mu \Sigma A\Sigma^\dagger L_\mu\right) , \\
Q_3^{PQA} &= \frac{\beta_3^{(8,8)}}{(4\pi)^2}\,\mathrm{str}\left(\Lambda\{\Sigma A\Sigma^\dagger, X_+\}\right) ,
\end{aligned}
\tag{4.1}
$$

where we introduced the $O(p^2)$ LECs $\beta^{(8,8)}_{1,2,3}$. For the PQ $K \to \pi$ matrix element, with degenerate valence quark masses ($M^2 = M_K^2 = M_\pi^2 = 2B_0 m_v$), we find at order p^2

$$
[K^+ \to \pi^+]^{QCD}_{\rm penguin} = \frac{4M^2}{f^2}\left\{\alpha_1^{(8,1)} - \alpha_2^{(8,1)} - \frac{2}{(4\pi)^2}\left(1 - \frac{K}{N}\right)\left(\beta_1^{(8,8)} + \frac{1}{2}\beta_2^{(8,8)} + \beta_3^{(8,8)}\right)\right\} . \tag{4.2}
$$

In this case the non-analytic terms coming from eq. (3.2) happen to vanish, so that only contributions from eq. (4.1) show up at this order, in addition to the tree-level terms coming from eq. (3.1) (there are, however, chiral logarithms coming from eq. (3.2) in the case $M_K \neq M_\pi$). The $K \to 0$ matrix element with non-degenerate valence quarks and non-degenerate sea quarks is

$$
\begin{aligned}
[K^0 \to 0]^{QCD}_{\rm penguin} = \frac{4i}{f}\Bigg\{ & \left(\alpha_2^{(8,1)} + \frac{2}{(4\pi)^2}\left(1 - \frac{K}{N}\right)\beta_3^{(8,8)}\right)(M_K^2 - M_\pi^2) + \frac{\alpha^{(8,8)}}{(4\pi)^2} \times \\
& \times \Bigg(\sum_{i\ valence} M_{3vi}^2 (L(M_{3vi}) - 1) - \sum_{i\ valence} M_{2vi}^2 (L(M_{2vi}) - 1) - \\
& - \sum_{i\ sea} M_{3si}^2 (L(M_{3si}) - 1) + \sum_{i\ sea} M_{2si}^2 (L(M_{2si}) - 1)\Bigg)\Bigg\} .
\end{aligned}
\tag{4.3}
$$

Here

$$
L(M) = \log \frac{M^2}{\Lambda^2} , \tag{4.4}
$$

and the result is given in the $\overline{MS}$ scheme, with Λ the running scale. M_{3si} (M_{2si}) is the mass of a meson made out of the 3rd (2nd) valence (i.e. the strange (down)) quark and the ith sea quark; M_{3vi} (M_{2vi}) is the mass of a meson made out of the 3rd (2nd) valence (i.e. the strange (down)) quark and the ith valence quark. At the order we are working, $M_{3si}^2 - M_{2si}^2 = M_{3vi}^2 - M_{2vi}^2 = M_K^2 - M_\pi^2$, which follows from $M_{3si}^2 = B_0(m_{v3} + m_{si})$, etc.

[5] One should also consider total-derivative terms like $i\partial_\mu \mathrm{str}\left(\Lambda[\Sigma A\Sigma^\dagger, L_\mu]\right)$ (the only one at order p^2), which however does not contribute to $K \to 0$ neither to $K \to \pi$ in the mass non-degenerate case $M_K \neq M_\pi$.

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From these results we learn several things. First, the contributions from the new operators to eqs. (4.2) and (4.3) indeed appear at order p^2, i.e. the same order as the leading order in the unquenched theory. Notice that in general, as e.g. in eq. (4.3), they contribute to a weak matrix element at leading order with non-analytic terms of the form $M^2 \log M^2$, which are absent in the unquenched case. Second, as one would expect, these results also contain the unquenched result as a particular case. This can be seen by choosing the number of sea quarks equal to the number of valence quarks (i.e. $N = K = 3$), and by equating corresponding quark masses, $m_{si} = m_{vi}$, $i = 1, \ldots, K$. For this choice, the terms proportional to $\alpha^{(8,8)}$ and $\beta^{(8,8)}_{1,2,3}$ in both matrix elements vanish, as they should. That this is also true at higher orders, as well as for other matrix elements, can be deduced from a quark-flow argument. For $N = K$, the first K entries in A (eq. (2.2)) vanish, leaving only sea and ghost quarks in the second factor $(\mathrm{str}\,(A\psi\overline{\psi}\gamma_\mu P_R))$ of $Q^{PQA}_{\rm penguin}$. If there are only valence quarks on the external lines, these sea and ghost quarks have to produce loops, which cancel if $N = K$ and their masses are pairwise equal. Note that the choice $N = K$ is necessary, because only in that case does the number of quarks in $(\overline{q}Xq)_R$ in eq. (1.1) correspond to the number of sea quarks.

At this point it is interesting to note that, although the terms proportional to $\alpha^{(8,8)}$ and $\beta^{(8,8)}_{1,2,3}$ are an unexpected "contamination", they still contain physical information about EM penguin matrix elements. There is also a way to avoid this contamination, by considering instead $Q^{PQS}_{\rm penguin}$ alone. In practice, this implies throwing out all Wick contractions in which q and $\overline{q}$ in eq. (1.1) (with $X = \mathbf{1}$) are contracted, except when they correspond to sea quarks.

Quenched ($N = 0$) results are obtained by replacing $\alpha^{(8,1)}_{1,2} \to \alpha^{(8,1)}_{q1,2}$, $\alpha^{(8,8)} \to \alpha^{NS}_q$ and $\beta^{(8,8)}_{1,2,3} \to \beta^{NS}_{q1,2,3}$ in eqs. (4.2), (4.3), and by dropping all terms containing sea quarks. One obtains

$$[K^+ \to \pi^+]^{QCD}_{\rm penguin} = \frac{4M^2}{f^2}\left\{\alpha^{(8,1)}_{q1} - \alpha^{(8,1)}_{q2} - \frac{1}{(4\pi)^2}(\beta^{NS}_{q1} + \frac{1}{2}\beta^{NS}_{q2} + \beta^{NS}_{q3})\right\}, \quad (4.5)$$

$$\begin{aligned}[K^0 \to 0]^{QCD}_{\rm penguin} = \frac{4i}{f}\Bigg\{ & \left(\alpha^{(8,1)}_{q2} + \frac{1}{(4\pi)^2}\beta^{NS}_{q3}\right)(M_K^2 - M_\pi^2) + \frac{\alpha^{NS}_q}{(4\pi)^2} \times \\ & \times \Bigg(\sum_{i\ valence} M^2_{3vi}(L(M_{3vi}) - 1) - \\ & - \sum_{i\ valence} M^2_{2vi}(L(M_{2vi}) - 1)\Bigg)\Bigg\}. \end{aligned} \quad (4.6)$$

In this case the terms proportional to α^{NS}_q and $\beta^{NS}_{q1,2,3}$ are a genuine contamination of the tree-level results coming from $Q^{QS}_{\rm penguin}$, and do not carry any physical information about EM penguins. Hence there is no reason to expect that a quenched lattice computation of a Q_5 or Q_6 matrix element has anything to do with the real world.

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Again, one may consider only $Q^{QS}_{\rm penguin}$ in order to determine $\alpha^{(8,1)}_{q1,2}$. This would mean that no "eye graphs" with q and $\overline{q}$ contracted would be considered at all.[6]

We conclude this section with the EM contributions, up to order p^2, to the matrix elements considered here (for the unquenched case see also ref. [14]). For the $K \to \pi$ matrix element with degenerate valence quarks and degenerate sea quarks we obtain from eq. (3.4)

$$[K^+ \to \pi^+]^{EM}_{\rm penguin} = 6\alpha^{(8,8)}\left\{1 - \frac{2N}{(4\pi f)^2} M^2_{VS}(L(M_{VS}) - 1)\right\}, \tag{4.7}$$

where $M^2_{VS} = B_0(m_v + m_s)$ is the (tree-level) mass-squared of a meson made out of one valence and one sea quark, and for the $K \to 0$ matrix element

$$\begin{aligned}[K^0 \to 0]^{EM}_{\rm penguin} = -2if\frac{\alpha^{(8,8)}}{(4\pi f)^2}\Big\{ & M^2_\pi L(M_\pi) - 2M^2_K L(M_K) + M^2_{33}L(M_{33}) + \\ & + \sum_{i\ sea}\left(M^2_{2si}(L(M_{2si}) - 1) - M^2_{3si}(L(M_{3si}) - 1)\right)\Big\}.\end{aligned} \tag{4.8}$$

The quenched result is obtained by dropping all terms containing the sea-quark mass, and by replacing $\alpha^{(8,8)} \to \alpha^{(8,8)}_q$.

In addition to the operators of eq. (4.1) with A replaced by Q, there are two more operators which contribute counterterms to eqs. (4.7), (4.8) at this order:[7]

$$\begin{aligned} Q^{PQA}_4 &= \frac{\beta^{(8,8)}_4}{(4\pi)^2}\,{\rm str}\,(\Lambda[\Sigma Q \Sigma^\dagger, X_-])\,, \\ Q^{PQA}_5 &= \frac{\beta^{(8,8)}_5}{(4\pi)^2}\,{\rm str}\,(\Lambda\Sigma Q\Sigma^\dagger)\,{\rm str}\,(X_+)\,. \end{aligned} \tag{4.9}$$

The EM counterterm contributions are

$$\begin{aligned} [K^+ \to \pi^+]^{EM}_{c.t.} &= \frac{4}{(4\pi f)^2}\Big((\beta^{(8,8)}_1 - \beta^{(8,8)}_2 + 7\beta^{(8,8)}_3 - 6\beta^{(8,8)}_4)M^2 + 3\beta^{(8,8)}_5 N M^2_{SS} - \\ & \qquad - 24\alpha^{(8,8)}(\lambda_5 M^2 + \lambda_4 N M^2_{SS})\Big), \qquad (4.10) \\ [K^0 \to 0]^{EM}_{c.t.} &= -\frac{4if}{(4\pi f)^2}\beta^{(8,8)}_3(M^2_K - M^2_\pi)\ , \end{aligned}$$

where the strong $O(p^4)$ LECs λ_i enter via wave-function renormalization and are related to the Gasser-Leutwyler L_i [12] by

$$\lambda_i = 16\pi^2 L_i\,. \tag{4.11}$$

Quenched results follow by setting $N = 0$ and replacing $\beta^{(8,8)}_i \to \beta^{(8,8)}_{qi}$, $i = 1, 2, 3, 4, 5$, $\lambda_k \to \lambda_{qk}$, $k = 4, 5$.

[6] The remaining "eye graphs" are those for which q ($\overline{q}$) is contracted with $\bar{s}$ (d) in eq. (1.1).

[7] The total-derivative term $i\partial_\mu{\rm str}\,(\Lambda[\Sigma Q\Sigma^\dagger, L_\mu])$ gives a contribution to $K \to \pi$ in the mass non-degenerate case $M_K \neq M_\pi$.

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5. Conclusion

In this paper we have considered the question as to what happens when one evaluates the matrix elements of QCD penguin operators in the quenched or partially quenched approximations which are commonly used in Lattice QCD. QCD penguins for $\Delta S = 1$ weak operators transform as an octet under $\mathrm{SU}(3)_L$ and as a singlet under $\mathrm{SU}(3)_R$. However, once one makes the transition from unquenched QCD, in which the weak hamiltonian at hadronic scales $\sim m_c$ is calculated, to quenched or partially quenched QCD, in which the lattice computations are done, the chiral group $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ enlarges to $\mathrm{SU}(K+N|K)_L \times \mathrm{SU}(K+N|K)_R$, and the corresponding statement is no longer true.

In the simple case of LR penguins considered here (cf. eq. (1.1)), the QCD penguins are a linear combination of two operators which transform in different irreps of the (partially) quenched symmetry group. The first of these transforms as a singlet under $\mathrm{SU}(K+N|K)_R$, much like in the unquenched case, and has a similar chiral behavior, with matrix elements linear in the quark masses at leading order in ChPT. The second operator, however, transforms in the adjoint representation (which generalizes the octet representation of SU(3)) of *both* $\mathrm{SU}(K+N|K)_L$ and $\mathrm{SU}(K+N|K)_R$. This new operator does not contribute at the lowest order in the chiral expansion to the matrix elements of interest, but it does contribute at next-to-leading order. However, because of the fact that this operator starts at order p^0 in (partially) quenched ChPT, its leading non-vanishing contributions compete with the leading contributions of the operators already present in the unquenched case.

At leading order in ChPT, the new operator corresponds to one new low-energy constant, $\alpha^{(8,8)}$ in the partially quenched case (cf. eq. (3.2)) and α_q^{NS} in the quenched case (cf. eq. (3.5)), while at higher orders new LECs proliferate, as usual. In the PQ case, these LECs turn out to be those corresponding to the EM penguin, because the new non-singlet operator and the EM penguin transform in the same irreducible representation of the enlarged chiral-symmetry group. In the quenched case, with no sea-quarks at all, this is not the case, and the new operators must be considered a pure quenching artifact. We have demonstrated the way the new operators work with the simple examples of $K \rightarrow$ *vacuum* and $K \rightarrow \pi$ matrix elements; leading-order expressions useful for practical applications can be found in section 4.

The implications of our analysis for the use of $K \rightarrow \pi$ and $K \rightarrow 0$ matrix elements in the determination of $K \rightarrow \pi\pi$ amplitudes are the following. To leading order in ChPT, the physical $K \rightarrow \pi\pi$ amplitude is determined by $\alpha_1^{(8,1)}$, and therefore this is the LEC one wishes to extract from the $K \rightarrow \pi$ matrix element. It is then clear that the way to do this is to consider only the singlet penguin, $Q^{PQS}_{\mathrm{penguin}}$, in the PQ theory with $N = 3$ light sea quarks. As already mentioned in section 4, this is equivalent to omitting all Wick contractions in which q and $\overline{q}$ in eq. (1.1) (with $X = \mathbf{1}$) are contracted, except when they correspond to sea quarks. Of course, since the $K \rightarrow \pi$

matrix element only gives the linear combination $\alpha_1^{(8,1)} - \alpha_2^{(8,1)}$, the $K \to 0$ matrix element of $Q_{\rm penguin}^{PQS}$ is also needed, as usual [11].

In the case that $N \neq 3$, one can still follow a similar strategy in order to determine $\alpha_1^{(8,1)}$. However, since the LECs depend on the number of light sea quarks N, there is no reason why the result should be the same as that of the real world, which has $N = 3$. In the quenched case ($N = 0$), the same strategy corresponds to considering only $Q_{\rm penguin}^{QS}$, or equivalently, no diagrams with q and $\overline{q}$ contracted at all. This would be the most reasonable strategy to adopt if one assumes that $\alpha_1^{(8,1)}(N = 0) \approx \alpha_1^{(8,1)}(N = 3)$. Another possibility, in the partially quenched case with $N \neq 3$ and in the quenched case, is to include both the singlet and non-singlet operators in the determination of $K \to \pi$ and $K \to$ *vacuum* matrix elements, and also in the conversion to $K \to \pi\pi$ amplitudes [10]. If one does include $q\overline{q}$ contractions for the valence quarks, the results will depend on the new LECs corresponding to the operators $Q_{\rm penguin}^{PQA}$ of the partially quenched case or $Q_{\rm penguin}^{QNS}$ of the quenched case and $Q_{1,2,3}$ of eq. (4.1). With either strategy, one cannot expect, *a priori*, to obtain a realistic result for the penguin contributions to $K \to \pi\pi$ amplitudes. In addition, since it is not known which of these strategies is a better approximation, the difference between them can be interpreted as an indication of the quenching error.

For LL penguin operators a similar analysis applies. When one replaces the second factor in eq. (1.1) by a left-handed current, $(\overline{q}Xq)_R \to (\overline{q}Xq)_L$, this factor is again not a singlet under $\mathrm{SU}(K + N|K)_L$, and new operators will appear. In this case however, we expect the new operator to start at order p^2 in ChPT. Since the lowest-order operators will again not contribute at tree level to matrix elements of interest, the new LL operators will only be relevant for a next-to-leading order analysis of lattice matrix elements [10].

Finally, we should emphasize that the effects of (partial) quenching on penguin operators discussed here are a property of the penguin operators themselves and not only of certain matrix elements, and will therefore have similar consequences for any weak matrix element to which such operators contribute, including not only non-leptonic kaon decays, but also non-leptonic B decays.

Acknowledgments

We would like to thank Tom Blum, Norman Christ, Bob Mawhinney, Amarjit Soni and other members of the RBC collaboration for asking questions leading to this paper. We also thank Claude Bernard, Santi Peris and Steve Sharpe for helpful discussions. MG would like to thank the particle theory groups of SISSA (Trieste, Italy) and the Universitat Autònoma (Barcelona, Spain) for hospitality. MG is supported in part by the US Dept. of Energy.

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